

Pionic Measures of Parity and \mathcal{CP} Violation in High Energy Nuclear Collisions

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The collisions of large nuclei at high energies could produce metastable vacua which are odd under parity, \mathcal{P} , charge conjugation, \mathcal{C} , and/or \mathcal{CP} . Using only the three-momenta of charged pions (or kaons), we show how to construct global observables which are odd under \mathcal{P} , \mathcal{C} , and \mathcal{CP} , and which can be measured on an event by event basis. Model dependent estimates of the \mathcal{P} -odd observables are on the order of 10^{-3} .

In a previous Letter we proposed a detailed model in which metastable vacua could arise in hot QCD [1]. These metastable states are related to θ -vacua, and so are odd under charge conjugation times parity, \mathcal{CP} . (For related studies, especially in supersymmetric theories, see [2–4].) An obvious question is how the breaking of this discrete symmetry by a metastable bubble could be measured in the collisions of large nuclei at high energies. As the bubble is odd under \mathcal{P} and \mathcal{CP} , the pions produced by its decay must also be in a state which is odd under these symmetries. In [1] we proposed measuring, on an event by event basis, a global variable which is odd under \mathcal{P} .

In this article we do two things. Using only the momenta of charged pions in an event, we construct global observables which are odd under the discrete symmetries of \mathcal{P} , \mathcal{C} , and/or \mathcal{CP} . Our discussion is general, independent of whatever detailed mechanism might produce nonzero values for these variables. For the collisions of nuclei with equal atomic number, as the initial state is even under \mathcal{P} , the observation of a \mathcal{P} -odd final state must be due to parity violation, such as by a \mathcal{P} -odd bubble. Based upon our specific model [1], we then give a rough estimate of the magnitude of the \mathcal{P} -odd and \mathcal{CP} -odd effects; we find that the asymmetries can be relatively large, at least $\sim 10^{-3}$.

At high energy, nucleus-nucleus collisions produce many pions, on the order of ~ 1000 per unit rapidity at RHIC energies. Experimentally, it is probably easiest to detect charged pions and their three-momenta. (All of our comments apply equally well to charged kaons.) Thus we are led to consider constructing observables only from the three-momenta of π^+ 's, \vec{p}_+ , and π^- 's, \vec{p}_- . As vectors, under parity the three-momenta transform as

$$\mathcal{P} : \vec{p}_+ \rightarrow -\vec{p}_+ , \quad \vec{p}_- \rightarrow -\vec{p}_- . \quad (1)$$

Charge conjugation switches π^+ and π^- ,

$$\mathcal{C} : \vec{p}_+ \leftrightarrow \vec{p}_- . \quad (2)$$

It is a theorem that any \mathcal{P} -odd invariant formed from three-vectors can be represented as a sum of terms, each of which involves one antisymmetric epsilon tensor [5]. The variable which we proposed previously is of this type [1]:

$$\mathbf{J} = \sum_{\pi^+, \pi^-} (\hat{p}_+ \times \hat{p}_-) \cdot \hat{z} . \quad (3)$$

In order to form \mathbf{J} we have introduced an arbitrary, fixed vector of unit norm, \hat{z} . If $\hat{z} \rightarrow -\hat{z}$ under parity, then \mathbf{J} is odd under \mathcal{P} . In \mathbf{J} we use the unit vectors $\hat{p}_\pm = \vec{p}_\pm / |\vec{p}_\pm|$ so that it is a pure, dimensionless number. The variable \mathbf{J} is separately odd under \mathcal{P} and \mathcal{C} , and so is even under \mathcal{CP} .

The variable \mathbf{J} is closely analogous to “handedness”, originally introduced to study spin dependent effects in jet fragmentation [6]. There the axis \hat{z} is usually defined by the thrust of the jet, with \hat{p}_+ and \hat{p}_- representing the directions of pions formed in the fragmentation of the jet. Correlations between the handedness of different jets produced in a given event are sensitive to \mathcal{CP} -violating effects [7].

It is not difficult to construct other invariants with different transformation properties. We introduce \vec{k}_\pm as

$$\vec{k}_\pm = \sum_{\pi^+} \vec{p}_+ \pm \sum_{\pi^-} \vec{p}_- , \quad \hat{k}_\pm = \vec{k}_\pm / k_\pm , \quad (4)$$

and then form

$$\mathbf{K}_\pm = \sum_{\pi^+, \pi^-} (\hat{p}_+ \times \hat{p}_-) \cdot \hat{k}_\pm . \quad (5)$$

The variables \mathbf{K}_\pm are \mathcal{P} -odd; \mathbf{K}_+ is \mathcal{C} -odd, and so \mathcal{CP} -even, while \mathbf{K}_- is \mathcal{C} -even, and so \mathcal{CP} -odd. The vector \vec{k}_+ measures the net flow of the charged pion momentum, while \vec{k}_- measures the net flow of charge from pions.

We can also form

$$\mathbf{L} = \sum_{\pi^+, \pi^-} (\hat{p}_+ \times \hat{p}_-) \cdot \hat{z} \left(\frac{p_+^2 - p_-^2}{p_+^2 + p_-^2} \right) . \quad (6)$$

The variable \mathbf{L} is \mathcal{P} -odd, \mathcal{C} -even, and \mathcal{CP} -odd. It does not require a net flow of momentum or charge to be nonzero, although as for \mathbf{J} , we do need to introduce an arbitrary unit vector \hat{z} .

Similar to \mathbf{L} , we can form [10]

$$\mathbf{M} = \sum_{\pi^+, \pi^-} \left(\frac{p_+^2 - p_-^2}{p_+^2 + p_-^2} \right). \quad (7)$$

This variable is \mathcal{P} -even, \mathcal{C} -odd, and so \mathcal{CP} -odd.

Besides using the vectors \vec{k}_\pm , another way of avoiding the introduction of an arbitrary unit vector \hat{z} is to use ordered pairs of pion momenta [8,9]; this procedure is also used in the studies of spin-dependent effects in jet fragmentation [6,7]. For any given pair of like sign pions, let \vec{p}'_+ denote the π^+ with largest momentum, so $|\vec{p}'_+| > |\vec{p}_+|$. This ordering is done in the frame in which the observable is measured. Then we can form a triple product as

$$\mathbf{T}_\pm = \sum_{\pi^+, \pi^-} (\hat{p}_+ \times \hat{p}_-) \cdot (\hat{p}'_+ \pm \hat{p}'_-) \quad , \quad |\vec{p}'_\pm| > |\vec{p}_\pm| \quad (8)$$

The variables \mathbf{T}_\pm are \mathcal{P} -odd; \mathbf{T}_+ is \mathcal{C} -odd and \mathcal{CP} -even, while \mathbf{T}_- is \mathcal{C} -even and \mathcal{CP} -odd. Besides the variables \mathbf{T}_\pm , one can clearly construct other \mathcal{P} - and \mathcal{C} -odd observables from triplets, or higher numbers, of pions.

The metastable \mathcal{P} -odd bubbles of ref. [1] were derived within the context of a nonlinear sigma model, with a $U(3)$ matrix U , $U^\dagger U = \mathbf{1}$. The metastable vacua are stationary points with respect to the nonlinear sigma model action, including the terms with two derivatives, a mass term, and an anomaly term [11]:

$$\mathcal{L} = f_\pi^2 \left\{ \frac{1}{2} \text{tr} (\partial_\mu U^\dagger \partial_\mu U) + c \text{tr} (M(U + U^\dagger)) - a(\text{tr} \ln U - \theta)^2 \right\}, \quad (9)$$

where $f_\pi = 93 \text{ MeV}$ is the pion decay constant, and θ is the θ -parameter. In QCD , $\theta \leq 10^{-9}$; we retain θ for now, because it helps illuminate the nature of the \mathcal{P} -odd bubbles. M is the mass matrix for current quark masses, $M = \text{diag}(m_1, m_2, m_3)$, and c is a constant. In this paper we only need the ratios of these quantities: taking $m_1 = m_u$, $m_2 = m_d$, $m_3 = m_s$, for the up, down, and strange quark masses, respectively, we take $m_u/m_d \approx 1/2$ and $m_u/m_s \approx 1/36$. The anomaly term is proportional to the topological susceptibility, $a \sim \int d^4x \langle Q(x)Q(0) \rangle$, where Q is the topological charge density, $Q = g^2/(32\pi^2) \text{tr}(G_{\mu\nu} \tilde{G}^{\mu\nu})$ [11]. At zero temperature, a is large, and the η' meson is heavy, $m_{\eta'}^2 \sim a$.

We are interested in stationary points of the effective lagrangian, \mathcal{L} . In our previous work [1], for simplicity we assumed that the stationary point is constant in space and time, so that only the mass and anomaly terms enter into the equations of motion. By a global chiral rotation,

a constant U field can be rotated into a diagonal matrix. With $U_{ij} = \exp(i\phi_i) \delta^{ij}$, the effective potential becomes

$$V(\phi_i) = f_\pi^2 \left(-c \sum_i m_i \cos(\phi_i) + \frac{a}{2} \left(\sum_i \phi_i - \theta \right)^2 \right), \quad (10)$$

At a stationary point, the ϕ_i 's satisfy

$$c m_i \sin(\phi_i) = a(\phi_1 + \phi_2 + \phi_3 - \theta). \quad (11)$$

We are interested in solutions for which the $\phi_i \neq 0$. It is clear from the stationary point condition that nonzero values of $\phi_1 + \phi_2 + \phi_3$ act like having a system with $\theta \neq 0$. With this insight, we set $\theta = 0$ henceforth.

As the anomaly term in (10) arises from $\text{tr} \ln U$, $\sum_i \phi_i$ is defined to be periodic modulo 2π . Consequently, when the anomaly term vanishes, $a = 0$, then any ϕ_i equal to a multiple of 2π is a solution, but by periodicity these are all equivalent to the trivial solution, $\phi_i = 0$. When $a \neq 0$, however, Witten observed [12] that there may be nontrivial solutions, in which some ϕ_i are *near* a multiple of 2π . These are not equivalent to the trivial vacuum, and represent the spontaneous breaking of \mathcal{P} and \mathcal{CP} symmetries, in exactly the same way as $\theta \neq 0$ violates \mathcal{CP} symmetry. These solutions only arise when a is sufficiently small. Based upon an analysis in the limit of a large number of colors, we suggested that near the phase transition, a becomes much smaller than its value at zero temperature [1]. In a mean field type of analysis, with T_d the temperature of the deconfining transition, and $t = (T_d - T)/T_d$ the reduced temperature, we found that $c(T) \sim 1/t^{1/2}$ and $a(T) \sim t$, so that the relevant ratio, a/c , scales as $a(T)/c(T) \sim t^{3/2}$. With the parameters of our model, we find that \mathcal{P} -odd bubbles appear once a/c is $\sim 1\%$ of its value at zero temperature. As $a/c \rightarrow 0$, these solutions satisfy $\phi_1 \approx 2\pi - \phi_u$, $\phi_2 \approx -\phi_d$, and $\phi_3 \approx -\phi_s$, where $m_u \phi_u \approx m_d \phi_d \approx m_s \phi_s$. For example, in our model, \mathcal{P} -odd bubbles first appear when $a/c < (a/c)_{cr} \sim .24$. At this point, $\phi_u \approx 1.8$, $\phi_d \approx .5$, and $\phi_s \approx .03$. We stress that this is only the stationary point with minimal action; for arbitrary integers $n = \pm 1, \pm 2, \dots$, there exist other stationary points with $\phi_1 \approx 2n\pi$, and $\phi_2 \approx \phi_3 \approx 0$, with energy densities which grow like $\sim n^2$.

In terms of the underlying gluonic fields, the \mathcal{P} -odd bubbles arise from fluctuations in the topological charge density, $G_{\mu\nu} \tilde{G}^{\mu\nu}$. It is easy to understand how a region in which $G_{\mu\nu} \tilde{G}^{\mu\nu} \neq 0$ can produce a \mathcal{P} -odd effect. Consider the propagation of a quark anti-quark pair through a region in which $G_{\mu\nu} \tilde{G}^{\mu\nu} \neq 0$; in terms of the color electric, \vec{E} , and color magnetic, \vec{B} , fields, $G_{\mu\nu} \tilde{G}^{\mu\nu} \sim \vec{E} \cdot \vec{B}$. If \vec{E} and \vec{B} both lie along the \hat{z} direction, then a quark is bent one way, the anti-quark the other, so that $(\vec{p}_q \times \vec{p}_{\bar{q}}) \cdot \hat{z} \neq 0$, where \vec{p}_q and $\vec{p}_{\bar{q}}$ are the three-momenta of the quark and anti-quark, respectively. While physically intuitive, this

picture does not allow us to directly relate this bending in the momenta of the quark and anti-quark to an asymmetry for charged pions.

To do so, we again resort to using an effective lagrangian. It is known that the effects of the axial anomaly show up in the effective lagrangians of Goldstone bosons in two [11–14], and only two [15], ways. One is through the anomaly term [11,12], $\sim a$, which we have already included. Besides that, however, the axial anomaly also manifests itself in the interactions of Goldstone bosons through the Wess-Zumino-Witten term [13,14]. This term is nonzero only when the fields are time dependent, which is why we could ignore it in discussing the static properties of \mathcal{P} -odd bubbles. It cannot be ignored, however, in computing the dynamical properties, and in particular the decay, of \mathcal{P} -odd bubbles. The Wess-Zumino-Witten term is manifestly chirally symmetric when written as an integral over five dimensions,

$$\mathcal{S}_{wzw} = -i \frac{1}{80\pi^2} \int d^5x \varepsilon^{\alpha\beta\gamma\delta\sigma} \text{tr} (R_\alpha R_\beta R_\gamma R_\delta R_\sigma) , \quad (12)$$

$R_\alpha = U^\dagger \partial_\alpha U$, but reduces to a boundary term in four space-time dimensions. For $U = \exp(iu)$, when $\partial_\alpha u \ll 1$,

$$\mathcal{S}_{wzw} \approx \frac{2}{5\pi^2} \int d^4x \varepsilon^{\alpha\beta\gamma\delta} \text{tr} (u \partial_\alpha u \partial_\beta u \partial_\gamma u \partial_\delta u) . \quad (13)$$

As discussed by Witten [14], the coefficient of the Wess-Zumino-Witten term is fixed by topological considerations, and is proportional to the number of colors, which equals three.

In a collision, we envision that the trivial vacuum heats up, a \mathcal{P} -odd bubble forms, and then decays as the vacuum cools. Since this represents bubble formation and decay, there is no net change in any topological number. Therefore, it is possible for a given event to contain an excess of bubbles over anti-bubbles (or vice versa), and thus to manifest true parity violation on an event by event basis.

To estimate the magnitude of the Wess-Zumino-Witten term for a \mathcal{P} -odd bubble, and to understand its effect on pion production, in (13) we can take three u 's to be condensate fields, $u \sim \phi_{u,d,s}$, and two to be charged pion fields, $u \sim \pi_\pm/f_\pi$. Suppose that the \mathcal{P} -odd bubble is of size R , with unit normal \hat{r} to the surface, and lasts for some period of time. Because of the antisymmetric tensor in (13), all three components of the condensate field must enter. Schematically, we obtain

$$\mathcal{S}_{wzw} \approx \frac{2}{5\pi^2} \int dt \int d^3r \phi_u \partial_r \phi_d \partial_0 \phi_s (\vec{p}_{\pi^+} \times \vec{p}_{\pi^-}) \cdot \hat{r} \quad (14)$$

The time integral is $\int dt \partial_0 \phi_s \sim \delta \phi_s = \phi_s$, since $\phi_s = 0$ in the normal vacuum. Similarly, the spatial integral is $\int d^3r \partial_r \phi_d (\vec{p}_{\pi^+} \times \vec{p}_{\pi^-}) \cdot \hat{r} \sim \int d\Omega \int R^2 dr \partial_r \phi_d (\vec{p}_{\pi^+} \times \vec{p}_{\pi^-}) \cdot \hat{r}$

$\sim \int d\Omega R^2 \phi_d (\vec{p}_{\pi^+} \times \vec{p}_{\pi^-}) \cdot \hat{r}$, where $\int d\Omega$ represents an integral over the direction of the normal, \hat{r} . Further, as the average momentum within the condensate is of order $p_\pi \sim 1/R$, the size of the bubble drops out as well. We thus obtain a final result which is independent of the size of the bubble, its lifetime, and its width:

$$\mathcal{S}_{wzw} \approx \frac{2\phi_u \phi_d \phi_s}{5\pi^2} \int d\Omega (\hat{p}_{\pi^+} \times \hat{p}_{\pi^-}) \cdot \hat{r} \quad (15)$$

We stress that it is only the decay of the \mathcal{P} -odd bubble which produces an effect, since the Wess-Zumino-Witten term vanishes for a static field. Also, note that there is only an observable asymmetry when $\phi_s \neq 0$; this is because in the absence of external gauge fields, there is only a Wess-Zumino-Witten term for three, and not for two, flavors. Within this model, \mathcal{S}_{wzw} is of similar form for two charged kaons.

Using our previous estimates for the ϕ 's, $\phi_u \sim \phi_d \sim 1$ and $\phi_s \sim 10^{-2}$, we obtain an effect of order $\sim 10^{-3}$. At the point where the \mathcal{P} -odd bubble first appears, $(a/c)_{cr}$, one can estimate that the energy density within the bubble, relative to the ordinary vacuum, is $\sim 25 n^2 \text{ MeV}/fm^3$, where n is the winding number of the bubble, $n = 1, 2, 3, \dots$. For a bubble $\sim 5 fm$ in radius, there are about $\sim 100 n^2$ pions produced in the decay of the bubble. If a fraction of the produced pions are observed within a given kinematical window, and we assume that all observed pions come from a portion of the total bubble, then we recover the variable \mathbf{J} , introduced before in (3), and find an estimate of $\mathbf{J} \sim 10^{-3}$. Moreover, we find a natural interpretation of the direction \hat{z} , which was needed to define \mathbf{J} , as the normal to the bubble's surface. One might wonder if the effect is diluted by the necessity to average over uncorrelated pairs. This does not happen, however, because the pion field within the bubble is a classical field, so that all charged pions are affected similarly.

Naively, one might expect that \mathbf{J} would average to zero over a single bubble. As the bubble is topological, though, the direction in which charged pions are swept is correlated with the sign of the condensate, so that a single \mathcal{P} -odd bubble can produce an effect in $\mathbf{J} \sim 10^{-3}$. Thus it is possible to distinguish between events in which bubbles are produced, and those in which bubbles are not, by measuring \mathbf{J} .

At first it may seem surprising that our \mathcal{P} -odd, \mathcal{C} -even, and \mathcal{CP} -odd bubble produces a signal in \mathbf{J} , which by previous analysis is \mathcal{P} -odd, \mathcal{C} -odd, and \mathcal{CP} -even. The Wess-Zumino-Witten term is even under parity, which is $\mathcal{P}_0(-1)^{N_B}$, where \mathcal{P}_0 is the operation of space reflection, and N_B counts the number of Goldstone bosons [14]. By scattering off a \mathcal{P} -odd bubble, we bring in an odd number of condensate fields, $\mathbf{J} \sim \phi_u \phi_d \phi_s$, (15), and so change the (apparent) quantum numbers to be \mathcal{P} -odd and \mathcal{CP} -even. This is only apparent, as scattering off an anti-bubble will give the opposite sign of \mathbf{J} .

We expect that bubbles will generate signals for the other variables presented of similar magnitude. For example, a single bubble will induce a net flow of pion charge, and so contribute to $\mathbf{K}_- \sim 10^{-3}$, (5). Through coherent scattering in a bubble, we would also expect the variables \mathbf{K}_+ , \mathbf{L} , and T_{\pm} to develop signals $\sim 10^{-3}$. Further, hot gauge theories can also exhibit metastable states which are \mathcal{P} -even and \mathcal{C} -odd [16]; these generate signals for \mathbf{M} [10].

The idea of exciting metastable vacua in hadronic reactions is an old one [17], as is the idea that a collective pion field can produce large fluctuations in heavy ion collisions on an event by event basis [18]. We wish to emphasize that there are certain topologically nontrivial configurations of pion fields which produce signals that are odd under the discrete symmetries of \mathcal{P} , \mathcal{C} , and/or \mathcal{CP} . Any observation of such violation of these discrete symmetries would be, *prima facie*, evidence of novel physics. Whatever credence one ascribes to our detailed dynamical model, the observables which we propose herein are possible to measure [19], and we strongly encourage our experimental colleagues to do so.

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